العمليات في مجموعة الأعداد الحقيقية

$$A=\frac{\sqrt{5}+1}{\sqrt{5}-3}+\frac{\sqrt{5}+3}{\sqrt{5}-1}=\frac{(\sqrt{5}+1)(\sqrt{5}-1)+(\sqrt{5}+3)(\sqrt{5}-3)}{(\sqrt{5}-3)(\sqrt{5}-1)}$$

$$=\frac{5-\sqrt{5}+\sqrt{5}-1+5-3\sqrt{5}+3\sqrt{5}-9}{5-\sqrt{5}-3\sqrt{5}+3}=0$$

$$B=\frac{2\sqrt{5}}{\sqrt{5}-2}-\frac{\sqrt{5}}{\sqrt{5}+2}=\frac{2\sqrt{5}(\sqrt{5}+2)-\sqrt{5}(\sqrt{5}-2)}{(\sqrt{5}-2)(\sqrt{5}+2)}=\frac{10+4\sqrt{5}-5+2\sqrt{5}}{5-2\sqrt{5}+2\sqrt{5}-4}=5+6\sqrt{5}$$

$$C=\frac{\sqrt{3}}{\sqrt{3}+2}+\frac{2\sqrt{3}}{\sqrt{3}+1}=\frac{\sqrt{3}(\sqrt{3}+1)+2\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}+2)(\sqrt{3}+1)}$$

$$=\frac{3+\sqrt{3}+6+2\sqrt{3}}{3+2\sqrt{3}+\sqrt{3}+2}=\frac{9+3\sqrt{3}}{5+3\sqrt{3}}=\frac{(9+3\sqrt{3})(5-3\sqrt{3})}{(5+3\sqrt{3})(5-3\sqrt{3})}$$

$$=\frac{45-27\sqrt{3}+15\sqrt{3}-27}{25-15\sqrt{3}+15\sqrt{3}-27}=\frac{18-12\sqrt{3}}{-2}=6\sqrt{3}-9$$
