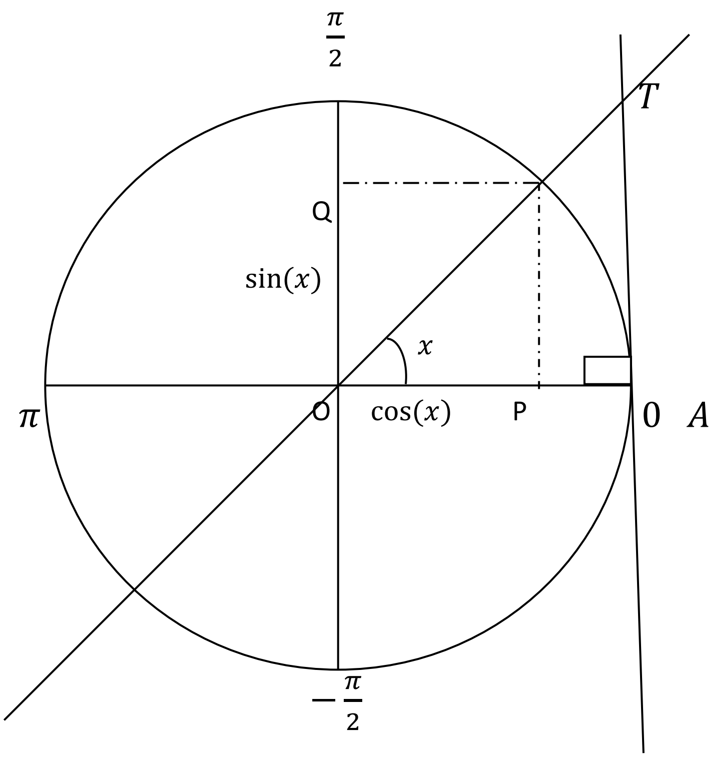
Définition

- $\cos x = \overline{OP}$ *et* $\sin x = \overline{OQ}$
- $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{\overline{OQ}}{\overline{OP}} = \overline{AT}$
- $Cotan x = \dfrac{cos x}{sin x} = \dfrac{1}{tan x}$



Conséquences

- $-1 \leq \cos x \leq 1$

  $-1 \leq \sin x \leq 1$
- $\cos(x + 2k\pi) = \cos x$

  $\sin(x + 2k\pi) = \sin x$
- $\cos^2 x + \sin^2 x = 1$
- $1 + \tan^2 x = \frac{1}{\cos^2 x}$

Angles associés

- $\cos(-x) = \cos x$ *et* $\sin(-x) = -\sin x$
- $\tan(-x) = -\tan x$

- $\cos(\pi + x) = -\cos x$ *et* $\sin(\pi + x) = -\sin x$
- $\cos(\pi - x) = -\cos x$ *et* $\sin(\pi - x) = \sin x$
- $\tan(x + \pi) = \tan x$ *et* $tan(\pi - x) = -tan x$

- $\cos\left(\frac{\pi}{2} - x\right) = \sin x$ *et* $\sin\left(\frac{\pi}{2} - x\right) = \cos x$
- $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$ *et* $\sin\left(\frac{\pi}{2} + x\right) = \cos x$
- $\tan\left(\frac{\pi}{2} - x\right) = \text{cotan}\, x$ *et* $\tan\left(\frac{\pi}{2} + x\right) = -\text{cotan}\, x$

Formules d'additions

$$\begin{cases} \bullet\ Cos(a+b) = cosa\,.\,cosb - sina\,.\,sinb \\ \bullet\ Sin(a+b) = sina\,.\,cosb + cosa\,.\,sinb \end{cases}$$

$$\begin{cases} \bullet\ Cos(a-b) = cosa\,.\,cosb + sina\,.\,sinb \\ \bullet\ Sin(a-b) = sina\,.\,cosb - cosa\,.\,sinb \end{cases}$$

$$\begin{cases} \bullet\ Cos(2a) = cos^2 a - sin^2 a = 1 - 2\,sin^2 a = 2\,cos^2 a - 1 \\ \bullet\ Sin(2a) = 2\,sin\,a\,.\,cos\,a \end{cases}$$

$$\begin{cases} \bullet\ Cos(\,a) = cos^2 \frac{a}{2} - sin^2 \frac{a}{2} = 1 - 2\,sin^2 \frac{a}{2} = 2\,cos^2 \frac{a}{2} - 1 \\ \bullet\ Sin(\,a) = 2\,sin\,\frac{a}{2}\,.\,cos\,\frac{a}{2} \end{cases}$$

$$\bullet\ cos^2 a = \frac{1 + Cos(2a)}{2} \qquad \bullet\ sin^2 a = \frac{1 - Cos(2a)}{2}$$

Transformation de $a\,cosx + b\,sinx$

$$a\,cosx + b\,sinx = r\,cos(x - \varphi)$$

$\bullet\ r = \sqrt{a^2 + b^2}$ $\qquad \bullet\ cos\varphi = \frac{a}{r}$

$\bullet\ sin\varphi = \frac{b}{r}$

Equations de bases

$$\bullet\ \cos x = \cos\alpha\ ssi \begin{cases} x = \alpha + 2k\pi \\ \quad ou \\ x = -\alpha + 2k\pi \end{cases}$$

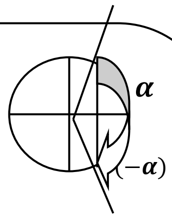
$$\bullet\ \sin x = \sin\alpha\ \ ssi \begin{cases} x = \alpha + 2k\pi \\ \quad ou \\ x = \pi - \alpha + 2k\pi \end{cases}$$

$$\bullet\ \tan x = \tan\alpha \quad ssi \quad x = \alpha + k\pi$$

Inéquations de bases

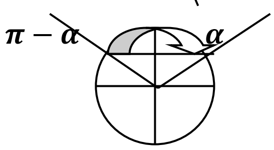
$\bullet\ \cos x \geq \cos\alpha$

$x \in [-\alpha + 2k\pi, \alpha + 2k\pi]$



$\bullet\ \sin x \geq \sin\alpha$

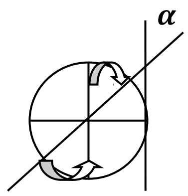
$x \in [\alpha + 2k\pi, \pi - \alpha + 2k\pi]$



$\bullet\ \tan x \geq \tan\alpha$

$x \in \left[\pi + \alpha + 2k\pi, -\frac{\pi}{2} + 2k\pi\right[$

$\cup \left[\alpha + 2k\pi, \frac{\pi}{2} + 2k\pi\right[$



coordonnées polaires

$\bullet\ M(x,y)$ coordonnées cartésiennes

$\bullet M(r,\theta)\ r > 0$

coordonnées polaires

$\bullet\ r = \sqrt{x^2 + y^2}$

$\bullet\ cos\theta = \frac{x}{r}$

$\bullet\ sin\theta = \frac{y}{r}$

EXERCICE N°1

$I)$ *calculer* $\cos\frac{5\pi}{4}$; $\sin(-\frac{2\pi}{3})$; $\cos\frac{5\pi}{6}$; $\sin 15\pi$; $\cos\frac{127\pi}{2}$ *et* $\sin(3000\pi)$

$II)$ *Soit* $f(x) = \cos 2x - \sin^2 x$

$a)$ *Calculer* $f(\frac{-15\pi}{6})$ *et* $f(5\pi)$

$b)$ *Montrer que* $f(x) = \dfrac{3\cos 2x - 1}{2}$

$c)$ *Montrer que* $\forall\, x \in \mathbb{R}$ *on a* $-2 \leq f(x) \leq 1$

$III)$ *Simplifier* $S = \cos(x + 20\pi) - \sin(\frac{9\pi}{2} - x) + 3\sin(x + \pi) + 6\cos\left(x - \frac{\pi}{2}\right)$

$T = \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7}$

$W = \sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \sin^2\frac{7\pi}{8}$

$R = \cos^2\frac{\pi}{5} + \cos^2\frac{3\pi}{10} + \cos^2\frac{4\pi}{5} + \cos^2\frac{7\pi}{10}$

EXERCICE N°2

1)*Soit* $x \in]0, \pi[$ *tel que* $\sin x = \frac{4}{5}$. *Calculer* $\cos x$ *et* $\tan x$

2)*Soit* $x \in \left]0, \frac{\pi}{2}\right[$ *tel que* $\tan x = \sqrt{3}$. *Calculer* $\cos x$, $\cos 2x$ *et* $\sin 2x$

EXERCICE N°3

Soient $f(x) = 1 - \cos 2x + \sin 2x$ *et* $g(x) = 1 + \cos 2x + \sin 2x$

1) $a/$*Montrer que* $f(x) = 2\sin x(\sin x + \cos x)$ *et que* $g(x) = 2\cos x(\sin x + \cos x)$

b/Montrer que $\sin x + \cos x = \sqrt{2}\cos(x - \frac{\pi}{4})$

c/En déduire $\sin\frac{\pi}{12}$

2)*Posons* $h(x) = \dfrac{f(x)}{g(x)}$.

Montrer que $h(x) = \tan x$. *En déduire* $\tan\frac{\pi}{12}$