

Identités remarquables (valables sur \mathbb{C})

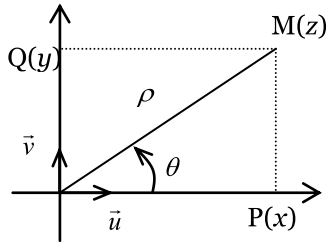
$$(a+b)^2 = a^2 + 2ab + b^2 ; (a-b)^2 = a^2 - 2ab + b^2 \qquad (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$a^2 - b^2 = (a+b)(a-b) ; a^2 + b^2 = (a+ib)(a-ib) \qquad (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Binôme de Newton : $(a+b)^n = a^n + C_n^1 a^{n-1} b + \dots + C_n^k a^{n-k} b^k + \dots + b^n$

Forme : algébrique $z = x + iy$

trigonométrique $z = \rho(\cos \theta + i \sin \theta) = \rho e^{i\theta}, \rho > 0$



$$\overline{OM} = x\bar{u} + y\bar{v}$$

$$OM = \rho = |z| = \sqrt{x^2 + y^2}$$

$$\overline{OP} = x = \operatorname{Re}(z) = \rho \cos \theta$$

$$\overline{OP} = y = \operatorname{Im}(z) = \rho \sin \theta$$

$$\tan \theta = \frac{y}{x}$$

Formules d'Euler

$$\cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$$

$$\sin \theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$$

Opérations algébriques

$$z + z' = (x + iy) + (x' + iy') = (x + x') + i(y + y')$$

$$z \cdot z' = (x + iy) \cdot (x' + iy') = (xx' - yy') + i(xy' + x'y)$$

Conjugué

$$z = x + iy = \rho e^{i\theta} ; \bar{z} = x - iy = \rho e^{-i\theta} ; \overline{z + z'} = \bar{z} + \bar{z}' ; \overline{zz'} = \bar{z} \cdot \bar{z}'$$

Inverse : $\frac{1}{z} = \frac{\bar{z}}{z\bar{z}} = \frac{\bar{z}}{|z|^2} = \frac{x}{x^2 + y^2} + i \frac{-y}{x^2 + y^2} = \frac{1}{\rho} e^{-i\theta} \quad x = \frac{1}{2}(z + \bar{z}) ; y = \frac{1}{2i}(z - \bar{z}) ; z\bar{z} = x^2 + y^2 = |z|^2$

Module et argument d'un produit, d'un quotient

$$z \cdot z' = (\rho e^{i\theta}) \cdot (\rho' e^{i\theta'}) = \rho\rho' e^{i(\theta+\theta')} \qquad |zz'| = |z| \cdot |z'| \qquad ; \arg(z \cdot z') = \arg(z) + \arg(z') [2\pi]$$

$$\frac{z}{z'} = \frac{\rho e^{i\theta}}{\rho' e^{i\theta'}} = \frac{\rho}{\rho'} e^{i(\theta-\theta')} \qquad \left| \frac{z}{z'} \right| = \frac{|z|}{|z'|} \qquad ; \arg\left(\frac{z}{z'}\right) = \arg(z) - \arg(z') [2\pi]$$

$$z^n = (\rho e^{i\theta})^n = \rho^n e^{in\theta}, n \in \mathbb{Z}$$

Distance de deux points : $AB = |b - a|$

Inégalité triangulaire :

$$\left| |z| - |z'| \right| \leq |z + z'| \leq |z| + |z'|$$

$$z \in \mathbb{R} \Leftrightarrow \operatorname{Im}(z) = 0 \Leftrightarrow \arg(z) = k\pi (k \in \mathbb{Z})$$

$$z \in i\mathbb{R} \Leftrightarrow \operatorname{Re}(z) = 0 \Leftrightarrow \arg(z) = \frac{\pi}{2} + k\pi (k \in \mathbb{Z})$$

Formule de Moivre : $\forall n \in \mathbb{N}^* , (e^{j\theta})^n = e^{jn\theta}$ ou $(\cos \theta + j \sin \theta)^n = \cos n\theta + j \sin n\theta$

Racines nièmes de l'unité : $u_k = e^{j \frac{2k\pi}{n}}$ où $k = 0, 1, 2, \dots, n-1$; $|u_k| = 1$

Les solutions de $z^n = a$, où $a = \rho e^{j\alpha}$, sont $z_k = z_0 u_k$, où $z_0 = \sqrt[n]{\rho} e^{j \frac{\alpha}{n}}$